Observational Constraints on Nfields Phantom Power-Law

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Abstract

In the multi-fields Phantom power law we investigate the analytical behavior of many scalar fields working collectively, where φ_i is the ith scalar field. Furthermore, we evaluate its parameter values by applying certain constraints on our model parameters, and then compare these values with current observational data from Observational data of CMB, BAO and H_0. Through our results, we deeply observe that in the dark-energy EOS parameter at the Big Rip always finite with the pressure and dark energy density divergence.

Keywords: phantom power-law Cosmology, multi-fields.

1 Introduction

Cosmological observations made, by end of the last century and beginning of this century, have conclusive evidence for late cosmic acceleration [1]. It is driven by an unknown fluid violating strong energy condition (SEC), such that $\omega < -\frac{1}{3}$, with ω being the ratio of pressure density to and energy density. This exotic fluid is known as dark energy differential equation (DE). Recently, Caldwell argued that experimental observations favor the case $\omega < -1$ more than the case $\omega > -1$ also disobeying the weak energy condition. Also the Dark energy, obeying $\omega < -1$, is dubbed as phantom [2]. With the use of General Relativity (GR)which based on Friedmann equations, it observed that phantom-dominated epoch of the universe goes faster, but ends up in the form of big-rip singularity in a finite future time [3]. Phantom dark energy fields are characterized by violating the main energy condition, $\rho + p > 0$. Also the conservation equation has the striking consequence that the energy density increases with expansion. When $p = \omega\rho$, with $\omega < -1$, are those condition when matter called Phantom energy [4]. On the basis of observational data R.R. Caldwell noted that EOS parameter "ω" has a very, short range near $\omega = -1$ with more likelihood to the side of $\omega < -1$. He argued that this possibility could not be neglected for the dark energy fluid. The scale satisfying the condition $\omega < -1$ is known as phantom. Alternatively, a very good description of the evolution of universe,is discussed in Reference [5, 6] and [7].

As energy density and pressure for a homogeneous scalar field is given by $\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$ and $p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$. For phantom scalar field φ , $w = \frac{p_\varphi}{\rho_\varphi}$, using the condition $w < -1$ in above equation, a surprising result is obtained that $\dot{\varphi}^2 < 0$, showing negative kinetic energy for the phantom scalar field. A power law cosmology is defined by the cosmological scale factor evolving as t^β [8].

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We want to bring down observational constraints on single field to extend this rule for Nfields power law. It is very interesting to generalize the above studies to multi-fields phantom power law with other potentials. For the simplicity of our model we take the isotropic and homogenous and Friedmann-Robertson-Walker (FRW) metric having curvature "k" and "t" is the cosmic proper time. In this paper, we study a general Nfiels power law cosmology with the scale factor given in terms parameter β with out any dimension [8]. In the case of Multi-fields , we modified the scale factor $a(t) = a_0((t_s - t)/(t_s - t_0))^\beta$ in order to achieve the self-stability, where t_s is a required positive reference time [9].

The field starts from near an unstable equilibrium (taken to be at the origin) and climb up the potential to a stable maxima. The Phantom model in which the observable Big Rip occur during the climb up of scalar field and its magnitude is at most of order M_p.

The arrangement of remaining sections of this paper is as; Section 2, we formulate the whole picture of phantom power-law cosmology for multi-fields. Section 3 we consider observational data to impose certain bounds on the Multi-fields parameters, and finally, Section 4 is devoted for summery and discussion.

2 Nfields with Power-Law Expansion

In this section we present Phantom cosmology under power law expansion, when many fields are working collectively with φ_i is the ith phantom scalar field. We assume the homogenous and isotropic Friedmann-Robertson Walker (FRW) background metric,

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d{\Omega_2}^2\right]. \tag{1}$$

Where Ω_2 is 2-dimension unit sphere volume, t is the cosmic time and k represents the curvature of 3-dimensional space with $k = 1, -1, 0$ corresponds to open, flat and closed universe respectively.
Our model is given by the following action, reads ([10],[11]):

$$S = \int d^4x\sqrt{-g}\sum_i\left[\frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i + V_i(\phi_i) + L_m\right]. \tag{2}$$

Where R the Ricci scalar, $V_i(\phi_i)$ is potential of phantom field and G the Newton gravitational constant. The Lagrangian L_m stands for the total matter of the universe including (dark plus baryonic). Finally, we concentrate on small redshifts, therefore, we are neglected the radiation sector, with speed of light as unity [12].

Through out our work we assume the flat geometry of the universe i.e. $k = 0$, for multi-field, the Friedmann equations write, reads ([13],[14])

$$H^2 \simeq \frac{1}{3M_p^2}\sum_i[\rho_{\phi_i} + \rho_{m_i}], \tag{3}$$

where $H(t) = \dot{a}/a$ is the Hubble parameter represents the expansion rate of the universe at time t and $M_p = (8\pi G)^{-\frac{1}{2}}$ is the Plank mass. In the above expression ρ_{ϕ_i} and p_{ϕ_i} are respectively the energy density and pressure of the ith phantom field which is denoted by ϕ_i, which are given by

$$\rho_\phi = \sum_i \rho_{\phi_i} = -\frac{1}{2}\sum_i \dot{\phi_i}^2 + \sum_i V_i(\phi_i). \tag{4}$$

$$p_\phi = \sum_i p_{\phi_i} = -\frac{1}{2}\sum_i \dot{\phi_i}^2 - \sum_i V_i(\phi_i). \tag{5}$$

Moreover, the evaluation equation for the multi-field is defined as

$$\sum_i \dot{\rho_{\phi_i}} + 3H\sum_i (p_{\phi_i} + \rho_{\phi i}) \simeq 0. \tag{6}$$

After simplification, the above equation becomes,

$$\sum_i \ddot{\phi_i} + 3H\sum_i \dot{\phi_i} - \sum_i \frac{dV_i}{d\phi_i} \simeq 0. \tag{7}$$

As in phantom cosmology the dark energy sector is attributed to the phantom fields, and thus its equation-of-state parameter is given by

$$\omega_m = \frac{\sum_i p_{m_i}}{\sum_i \rho_{m_i}}, \tag{8}$$

where $\omega_m = \sum_i \omega_{m_i}$.
For the matter density

$$\sum_i \dot{\rho_{m_i}} + 3H\sum_i (1+\omega_{m_i})\rho_{m_i} \simeq 0, \tag{9}$$

with the simple form of its solution is

$$\sum_i \rho_{m_i} = \sum_i \frac{\rho_{m_0}}{a^{n_i}} \tag{10}$$

where $n_i = 3(1+\omega_{m_i})$ and we are discussing only massive scalar fields, but the case of massless scalar fields are neglected here. As a special case when ω_{m_i} is equal to zero, then $n = 3$.
Also from Eq. (4)

$$\dot{H} \simeq \frac{1}{6M_p^2}\left[3\sum_i \dot{\phi}_i^2 - \sum_i \rho_{m_i} n_i\right]. \tag{11}$$

When we are working with Phantom cosmology, we replace t by $t_s - t$; the reference time t_s is sufficiently positive, then we obtain

$$a(t) = a_0\left(\frac{t_s - t}{t_s - t_0}\right)^\beta \tag{12}$$

with the Hubble parameter and its derivatives with respect to time is

$$H(t) \simeq \frac{-N\beta}{(t_s - t)} \tag{13}$$

Thus for the value of β less than zero, we find an accelerating "$\ddot{a}(t)$" is greater than zero and expanding "$\dot{a}(t)$" is greater than zero, universe, we find that $\dot{H}(t)$ is positive therein, which implies that it provides super acceleration, this is only possible for phantom power-law cosmology. In addition, for the exponent $\beta < 0$, and at late time $t = t_s$, the scale factor $a(t)$ and $H(t)$ of the universe both are diverge, as a result it goes to a Big Rip.

But then, the important point which is already discussed in reference [15], We find the potential $\sum_i V_i(t)$, by using Eqs.(3) and (4) in Eq. (11). Since $n_i = 3(1 + \omega_{m_i})$; $1 \le i \le N$. For dust dominated universe $\omega_{mi} \to 0$, this implies that $n_i = n = 3$ and as a result we obtain $\sum n_i = 3N$. Therefore

$$V(t) = \sum_i V_i(t) = N\left[M_p^2\left(\frac{3\beta^2 - \beta}{(t_s - t)^2}\right) - \frac{5}{6}\frac{\rho_{m_0}}{a_0{}^3}\left[\frac{t_s - t_0}{t_s - t}\right]^{3\beta}\right]. \tag{14}$$

From Eq. (11) we can obtain

$$\dot{\phi}^2(t) = N\left[\frac{-2\beta M_p^2}{(t_s - t)^2} + \frac{\rho_{m_0}}{a_0{}^3}\left(\frac{t_s - t_0}{t_s - t}\right)^{3\beta}\right], \tag{15}$$

where $\dot{\phi}(t)^2 = \sum_i \dot{\phi}_i^2$.
Using the values from Eqs. (14) and (15) in Eq. (4)

$$\rho_\phi = \sum_i \rho_{\phi i} \simeq N[M_p^2\frac{3\beta^2}{(t_s - t)^2} - \frac{4\rho_{m_0}}{3a_0{}^3}\left(\frac{t_s - t_0}{t_s - t}\right)^{3\beta}]. \tag{16}$$

Again putting Eqs. (14) and (15) into Eq.(5) we get

$$p_\phi \simeq N\left[M_p^2\frac{(-3\beta^2 - \beta + 3\beta)}{(t - t_s)^2} - (t - t_s)H(t) + \frac{\rho_{m0}}{3a_0{}^3}\left(\frac{t_s - t_0}{t_s - t}\right)^{3\beta}\right]. \tag{17}$$

In the case of phantom Nfields, the dark energy equation-of-state is

$$\omega_{DE}(t) = p_\phi/\rho_\phi$$

which implies that

$$w_{DE}(t) = (-1 + \frac{1}{\beta}) \tag{18}$$

We see that for Big Rip behavior $\omega_{DE}(t)$ always having finite value [16].

For β less than zero possesses additionally a positive $\dot{H}(t)$ that leads to super-acceleration [17]. So such kind of scenario expansion is always came with acceleration.
Furthermore, with the value of β less than zero, at t equal to t_s the scale factor and the Hubble parameter diverges, that is the universe results to a Big Rip. We investigate that these behaviors of Nfields phantom cosmology are very similar with result of single field phantom cosmology with power-law[15].

3 Observational Bound For Nfields With Model Parameters

In this section, we apply the techniques that conform the values of multi-fields with the observational data. Now we are fitting the observational data, presenting our results in the case of many fields work collectively. We observed that our values for all parameters in **Table I** are best fitted with minor error of accuracy, which is negligible for large scale, we also provide the 1σ bound of every parameter. Similarly in **Table II** we present the maximum possibility of the values up to 1σ bound for the derived parameters, namely the power-law exponent β, the present matter energy density value ρ_{m_0}, the present critical energy density value ρ_{c_0} and the Big Rip time t_s. As we observed that β is always less than zero, as expected in consistent phantom cosmology. Furthermore, we observed that the BR time is one order of magnitude greater than the present age of the universe, which shows that such an outcome is predictable in phantom cosmology, unless one include additional mechanisms as shown in [19].

$$V(t) \approx N\left[\frac{6.5\times 10^{27}}{(3.30\times 10^{18}-t)^2} - 2.52\times 10^{-371}\times(3.30\times 10^{18}-t)^{19.54}\right]. \tag{19}$$

While when we consider $WMAP7$ data alone, it provides

$$V(t) \approx N\left[\frac{6.41\times 10^{27}}{(3.30\times 10^{18}-t)^2} - 1.98\times 10^{-369}\times(3.30\times 10^{18}-t)^{19.37}\right]. \tag{20}$$

Here we noted that in above results although the second term is very small at early times of the universe, but becomes very important at late times, this situation is close to the BR. Now the scalar field evolution at late time ($t\rightarrow t_s$), ρ_{m_0} can be neglected and also set $a_0 = 1$.
Then we obtain the new result

$$\varphi(t) = \int\sqrt{N\left[\frac{-2\beta M_p^2}{(t_s-t)^2}\right]}dt. \tag{21}$$

Furthermore, for the combined $WMAP7+BAO+H_0$, we get

$$\varphi(t) \cong -\sqrt{2N\beta M_p^2}\ln|t_s-t|, \tag{22}$$

which implies that

$$\varphi(t) \cong \sqrt{N}[-2.645\times 10^{13} * \ln|3.30\times 10^{18}-t|] \tag{23}$$

while single $WMAP7$ data provides that

$$\varphi(t) \cong \sqrt{N}[(-2.643\times 10^{13}) * \ln|3.30\times 10^{18}-t|] \tag{24}$$

According to our exactions the phantom field and the kinetic energy diverges at the Big Rip.

Parameter	$H_0+WMAP7+BAO$	WMAP7
t_0	$13.78\pm 0.11 Gyr[(4.33\pm 0.04)\times 10^{17} sec]$	$13.71\pm 0.13 Gyr[(4.32\pm 0.04)\times 10^{17} sec]$
H_0	$70.2^{-1.4}_{+1.3}\quad km/s/Mpc$	$71.4\pm 2.5\quad km/s/Mpc$
Ω_{b0}	0.0455 ± 0.0016	0.0445 ± 0.0028
Ω_{CDM0}	0.227 ± 0.014	0.217 ± 0.026

TABLE I: Observational maximum likelihood values in 1σ confidence level [18].

Parameter	$H_0 + WMAP7 + BAO$	WMAP7
β	$-6.52^{-0.25}_{+0.24}$	-6.51 ± 0.4
ρ_{m0}	$(-6.532 \pm 0.38) \times 10^{-27} \quad kg/m^3$	$(2.513 \pm 0.27) \times 10^{-27} \quad kg/m^3$
ρ_{c0}	$(9.4^{-0.4}_{+0.3}) \times 10^{-27} \quad kg/m^3$	$(9.63 \pm 0.58) \times 10^{-27} \quad kg/m^3$
t_s	$104.83^{-1.8}_{+2.1}\ Gyr \quad \left[(3.30 \pm 0.06) \times 10^{18} sec\right]$	$102.8 \pm 3.4\ Gyr \quad \left[(3.24 \pm 0.21) \times 10^{18} sec\right]$

TABLE II: Derived maximum likelihood values in 1σ confidence level for the power-law exponent.

4 Summary and Discussion

From Eq.(23) and Eq. (24), we find the different values of t and put them in Eq. (19) and Eq. (20), to evaluate the potential for the Nfields Phantom power-law i.e. $V(\varphi)$.

Thus, for the $WMAP7 + BAO + H_0$ and $WMAP7$, we obtain different values of $V(\varphi)$

$$\approx N[6.48 \times 10^{27} e^{\sqrt{N}(0.77\times10^{-13}\varphi)} - 2.52 \times 10^{-371} e^{\sqrt{N}(-7.44\times10^{-13}\varphi)}]$$

and

$$\approx N[6.39 \times 10^{27} e^{\sqrt{N}(0.78\times10^{-13}\varphi)} - 1.98 \times 10^{-369} e^{\sqrt{N}(-7.5\times10^{-13}\varphi)}]$$

respectively.

Through out our work we study the phantom cosmology for multi-fields, we observed that the cosmic scale factor $a(t)$ is obeying the power law. When we construct the whole scenario, we fit the observationally data of $WMAP7+BAO+H_0$ and $WMAP7$ alone by applying bound on the multi-fields by focusing on exponent β and the BR time t_s. By using separately WMAP7 data, we obtained the value $\beta \cong -6.523 \pm 0.38$, while the BR is observed at $t_s \cong 102.8 \pm 3.475\ Gyr$. However, the dark energy equation of state parameter ω_{DE} lies below the phantom divide, it was expected and at BR time it always remains finite and equal to -1.1533. Although the phantom dark anergy density and pressure are diverge at the BR. By using $WMAP7 + BAO + H_0$ data set alone we find $\beta \approx -6.51^{-0.25}_{+0.24}$, while the Big Rip is observed at $t_s \approx 104.5^{-2.0}_{+1.9}\ Gyr$, in 1σ confidence level. Definitely, the subject of Nfields quantization of such scenarios is open and needs further investigation on this.

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References

[1] S.J. Perlmutter et al., Astrophys. J. 517,565 (1999); D.N. Spergel et al, Astrophys J. Suppl. 148 175 (2003) and G. Calcagni, Phys. Rev. D 69, 103508, (2004).

[2] S.K. Srivastava, 0707.1376v6 [gr-qc], (2008).

[3] R.R. Caldwell,Phys. Lett. B 545, 23 (2002).

[4] A. Liddle "An Introduction to Modern Cosmology" (Second Edition), (2003).

[5] S. Nojiri and S.D. Odintsov, Gen. Rel. Grav. 38, 1285 (2006). I.P. Neupane and H. Trowland, 0902.1532 [gr-qc]; I. P. Neupane and C. Scherer, JCAP 0805, 009 (2008).

[6] E. W. Kolb, Astrophys. J. 344, 543 (1989).

[7] M. Jamil, A. Qadir, Gen.Rel.Grav.43, 1069-1082 (2011)

[8] A. Dev, D. Jain and D. Lohiya , 0804.3491v1 [astro-ph] (2008).

[9] S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. 38, 1285. (2006). I. P. Neupane and H. Trowland, 0902.1532 [gr-qc]; I. P. Neupane and C. Scherer, JCAP 0805, 009 (2008).

[10] R. R. Caldwell, Phys. Lett. B 545, 23 (2002); S. Nojiri and S. D. Odintsov, Phys. Lett. B 562, 147 (2003); P. Singh, M. Sami and N. Dadhich, Phys. Rev. D 68, 023522 (2003); J. M. Cline, S. Jeon and G. D. Moore, Phys. Rev. D 70, 043543 (2004); V. K. Onemli and R. P. Woodard, Phys. Rev. D 70, 107301 (2004); W. Hu, Phys. Rev. D 71, 047301 (2005); M. R. Setare and E. N. Saridakis, JCAP 0903, 002 (2009); E. N. Saridakis, Nucl. Phys. B 819, 116 (2009); S. Dutta and R. J. Scherrer, Phys. Lett. B 676, 12 (2009).

[11] Y.F. Cai W. Xue, Phys.Lett.B 680,395-398,(2009).

[12] R. R. Caldwell, Phys. Lett. B 545, 23 (2002); S. Nojiri and S. D. Odintsov, Phys. Lett. B 562, 147 (2003); P. Singh, M. Sami and N. Dadhich, Phys. Rev. D 68, 023522 (2003); J. M. Cline, S. Jeon and G. D. Moore, Phys. Rev. D 70, 043543 (2004); V. K. Onemli and R. P. Woodard, Phys. Rev. D 70, 107301 (2004); W. Hu, Phys. Rev. D 71, 047301 (2005); M. R. Setare and E. N. Saridakis, JCAP 0903, 002 (2009); E. N. Saridakis, Nucl. Phys. B 819, 116 (2009); S. Dutta and R. J. Scherrer, Phys. Lett. B 676, 12 (2009).

[13] I. Ahmad, Y.S. Piao, C.F. Qiao, JCAP 0802, 002 (2008).

[14] I. Ahmad, Y.S. Piao, C.F. Qiao, JCAP 0806, 023 (2008).

[15] C. Kaeonikhom , B. Gumjudpai and E.N. Saridakis,Phys. Lett. B 695, 45-54 (2011)

[16] S. Nojiri, S. D. Odintsov and S. Tsujikawa, Phys. Rev. D 71, 063004 (2005).

[17] S. Das, P. S. Corasaniti and J. Khoury, Phys. Rev. D 73, 083509 (2006); M. Kaplinghat and A. Rajaraman, Phys. Rev. D 75, 103504 (2007).

[18] E. Komatsu, et al. 1001.4538v3 [astro-ph], (2011).

[19] M. Sami and A. Toporensky, Mod. Phys. Lett. A 19, 1509 (2004);